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MODELING THE ACQUISITION AND ENGAGEMENT OF RELOCATABLE NUCLEAR TARGETS

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CAA

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The US Army Concepts Analysis Agency (CAA) is responsible for modeling the employment of tactical nuclear weapons at the theater level. Currently available simulations that model the exchange of tactical nuclear weapons require much time to set up and run. CAA has developed a simple probability model based on alternating renewal processes to provide a quick estimate of the probability that a relocatable combat unit can be acquired and engaged by nuclear weapons, without a loss in accuracy compared to the detailed simulation. The model may be applied to either tactical or strategic relocatable targets.

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June 1989

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MODELING THE ACQUISITION AND ENGAGEMENT OF RELOCATABLE NUCLEAR TARGETS

PAPER SUMMARY

ABSTRACT. The US Army Concepts Analysis Agency (CAA) is responsible for modeling the employment of tactical nuclear weapons at the theater level. Currently available simulations that model the exchange of tactical nuclear weapons require much time to set up and run. CAA has developed a simple probability model based on alternating renewal processes to provide a quick estimate of the probability that a relocatable combat unit can be acquired and engaged by nuclear weapons, without a loss in accuracy compared to the detailed simulation. The model may be applied to either tactical or strategic relocatable targets.

THE RESEARCH SPONSOR was the Director, US Army Concepts Analysis Agency.

THE OBJECTIVE OF THE RESEARCH was to develop an analytic model for the acquisition and movement of relocatable nuclear targets, in order to estimate the probabilities that target units may be acquired, be available for fire planning, and be available for engagement with a nuclear weapon.

THE MAIN ASSUMPTIONS used in this research were:

(1) For any target unit, the time to acquire it, the time that it is retained, the time that it stays in place and the time that it is moving are mutually independent random variables with stationary probability distributions.

(2) The acquisition process and movement process for any given unit are independent.

(3) Units are acquired and move independently of each other during the short timeframe of interest (generally less than 12 hours).

THE BASIC APPROACH used in this research was to model the acquisition process and the movement process for each unit as independent, stationary alternating renewal processes.

THE PRINCIPAL FINDING of the research is that is is possible to develop a model of the acquisition and movement state of relocatable nuclear targets using an alternating renewal process representation.

THE RESEARCH WAS PERFORMED BY MAJ Mark A. Youngren.

COMMENTS AND QUESTIONS may be sent to the Director, US Army Concepts Analysis Agency, ATTN: CSCA-RQN, 8120 Woodmont Avenue, Bethesda, MD 20814-2797.

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Introduction

The US Army Concepts Analysis Agency (CAA) performs analyses for the Department of the Army relating to engagement of armed forces at the theater level, including the tactical employment of nuclear weapons. When we examine issues relating to tactical nuclear warfare, we need to be able to determine the probability that we can acquire potential nuclear targets, retain them long enough to plan the nuclear fires, and have the target still be in place at the time that detonation occurs. In order to alter the course of the battle to meet theater-level or strategic objectives, nuclear weapons use may be constrained to achieve a specific purpose within an appropriate period of time. For example, weapons may not be fired at potential targets as they are acquired; there may be significant delays between the time the fires are planned and the time of detonation.

CAA currently uses the Nuclear Fire Planning and Assessment Model III (NUFAM III) (Schuetze and Albrecht [1986]), a detailed, stochastic simulation model of a two-sided tactical nuclear exchange, to support analyses of theater nuclear issues. The model is very useful in providing a representation of the effects of a postulated nuclear exchange, but it is very time- and resource-intensive to set up, run, and analyze the output. This situation generated a desire for a simple analytic model that can be used to quickly estimate the probability of engagement of potential nuclear targets without having to set up a series of simulation runs.

Our solution to this problem is a renewal-based probability model of acquisition and engagement of relocatable nuclear targets at the tactical or strategic level that can be used to quickly estimate the probability that targets can be acquired, planned for fire, and be available to be hit by a nuclear weapon. NUFAM III currently explicitly simulates independent movement and acquisition times using the same movement and acquisition distributions throughout the simulation -- thus, NUFAM is generating realizations of independent, stationary alternating renewal processes, and the probability model described herein provides a direct alternative to the estimation process internal to NUFAM. Of course, NUFAM III also models processes not addressed in this paper, such as weapon allocation and damage assessment. However, use of this renewal model in NUFAM

III will greatly increase the efficiency of the simulation, and use outside of NUFAM will permit a quick, identical estimate of the probabilities of acquisition and engagement without having to exercise the NUFAM simulation.

Modeling Theater-Level Tactical Nuclear Warfare

Modeling nuclear weapons at a theater level poses several problems, brought about by an asymmetry in the resolution of our representation of theater combat and the resolution required for modeling tactical nuclear effects. Combat at a theater level involves large areas of space and vast numbers of combat units, support forces, and militarily significant entities. Limitations brought about by available computer resources (even using supercomputers) and, more importantly, our capability to understand and properly represent such combat in a computer model, require significant simplification. The most important simplifications in theater level modeling involve the relatively low level of resolution of the models, and the use of deterministic (as opposed to stochastic) processes and data. We rely upon these low-resolution combat models or scenarios to provide the initial situation (force arrays, strengths, etc.) to our models of tactical nuclear warfare.

Several models of conventional warfare exist at the theater level. The model used at CAA is called the Force Evaluation Model (FORCEM). Like most theater-level models and scenarios, FORCEM simplifies by representing combat forces at the division and higher level and representing time in 12-hour time steps. Our models of tactical nuclear warfare such as NUFAM III, on the other hand, require a much higher degree of resolution in time and space. Combat units may be targeted at the company level and artillery units at the level of individual missile launchers. These units may move and be acquired multiple times within 12 hours. The period during which nuclear weapons are planned and fired is modeled on a time scale of minutes to hours, well within a 12-hour time period. As a result, we are uncertain about the precise location, movement and acquisition of these higher resolution units, and we rely on probability models to describe our uncertainty regarding them. Specifically, we need to represent the effects of acquisition and movement of potential target units within a period of time (e.g., 12 hours) during which the opposing force structures remain approximately the same (as represented in theater-level combat models).

Ideally, we would like to represent the detailed acquisition and movement processes, interdependent with conventional combat, the environment, etc. in detailed simulation or analytic probability models. As we have explained previously, this level of detail is not practical at the theater level. What we do, instead, is to run high resolution models of small force sizes and areas to derive estimates of the distributions of the acquisition and movement times of interest. These distributions are averaged over stochastic environments, terrain, etc. to derive summary estimated distributions for the acquisition and movement times of interest. These distributions will hold for at least the 12-hour period within the conventional model or scenario which defines the scenario, force structure, sensor capabilities, etc. for these high resolution models. In practice, these averaged distributions are used to describe our uncertainty about acquisition and movement over longer periods of time (24 to 36 hours). Models and scenarios other than FORCEM may have different resolution in time but the same principles apply.

The issue of primary interest in this paper is how we use this summary information to adequately represent the acquisition and movement processes during the time when tactical nuclear weapons are planned and fired. NUFAM III uses these distributions to generate an explicit sequence of acquisition and movement events for every unit represented in the model. We will show how this explicit simulation of acquisition and movement can be eliminated without a loss in relative modeling fidelity.

The Target Acquisition Process

Detecting target units with sufficient accuracy to plan for nuclear fires is the process of *target acquisition*. *Targets* are combat units or major items of equipment such as mobile missile launchers that are considered for engagement by nuclear weapons. *Relocatable targets* are targets which have the capability to move during the scenario of interest (although they may or may not retain mission capability during movement). As a consequence of this movement capability, targets do not remain acquired indefinitely (unless they can be tracked indefinitely once acquired); at some time, they move, and the acquisition is no longer valid. Even if a tracking capability exists, there is a probability that such tracking will be lost over time.

The outcome of the target acquisition process is a changing acquisition list. A target unit is *acquired* when it is detected by a sensor, identified as a target, and placed on the acquisition list. A target unit may be *dropped* from the list either due to a negative sensor report (i.e., we no longer detect its presence), or it may be dropped after some period of time when the acquisition information cannot be updated. Any given target unit will alternate between two states: acquired (retained on the list) or not acquired. A target acquisition process is therefore a temporal series of such acquisition states. The *time to acquisition*, T_a, is the time it takes to acquire a target once any previous acquisition has been dropped: the *time of retention*, T_r, is the time a target is retained on the acquisition list.

Targets that are to be engaged using conventional weapons are generally fired upon soon after acquisition. Nuclear targets differ from conventional as they are planned for specific purposes dictated by the overall tactical and/or strategic situation. As a result, they are not normally engaged as they are acquired; rather, nuclear fires are directed at targets that are acquired and perceived to be in place at the time the weapons are approved for fire.

Figure 1 illustrates a representative acquisition sequence for a relocatable target unit. Once the unit has been dropped from the list, it is immediately subject to being reacquired. We expect that nuclear weapons use will occur after the conventional battle has been underway for some time: thus, we are interested in the acquisition probabilities at some point in time well after the acquisition process has begun. We assume that the times to acquisition $\{T_a\}$ are independent and identically distributed (iid) and make the same assumption about the times of retention $\{T_r\}$. For relocatable targets, we can approximate the target acquisition process as an alternating renewal process of indefinite length. Both the time that the target is dropped from the list and the time that the target is acquired are renewal points of this alternating renewal process.

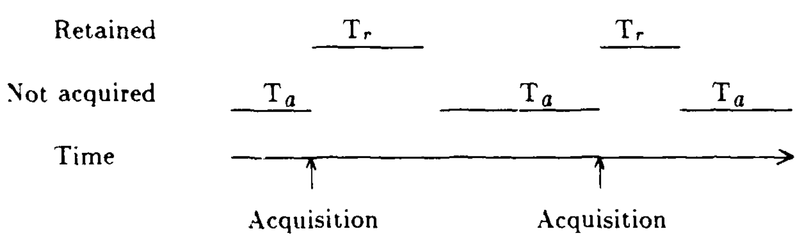

Figure 1. Possible Target Acquisition Sequence

The Fire Planning and Engagement Sequence

Enemy units are identified as potential nuclear targets as they are acquired. If the scenario calls for the use of tactical nuclear weapons, the model must be able to represent employment against specific types of targets that may be found during a limited period of time. Because of the constraints associated with nuclear weapons employment, each nuclear fire mission must be carefully planned before orders are given to fire the weapons. The fire planning process will normally terminate at the beginning of some specified time interval of length L within which weapons may be fired, which we refer to as the *fire execution period.* We identify the time that the fire planning process ends as t_p. A target unit that may be planned will have had a recent acquisition time A prior to t_p. The nuclear round will detonate at some time T_d during the fire execution cycle; generally, this time is specified during the planning but is not known *a priori* when modeling the engagement sequence (Figure 2).

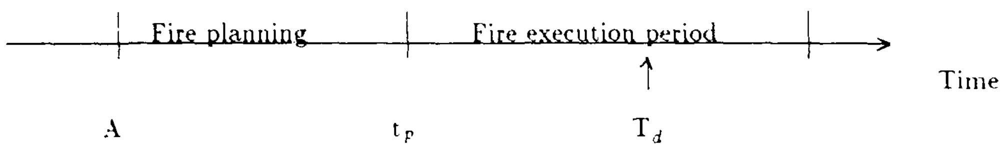

Figure 2. Acquisition, Planning, and Detonation Events for an Acquired Target

The important thing to note is that t_p is chosen independently of the target acquisition and movement sequences of any potential target unit. As a result, the time t_p may be viewed as a random entry point into these processes. Following the terminology common in reliability, we call the time from the last transition to our random entry point the *age* and the time from our random entry point to the next transition the *residual life.* An example using the acquisition sequence is shown in Figure 3.

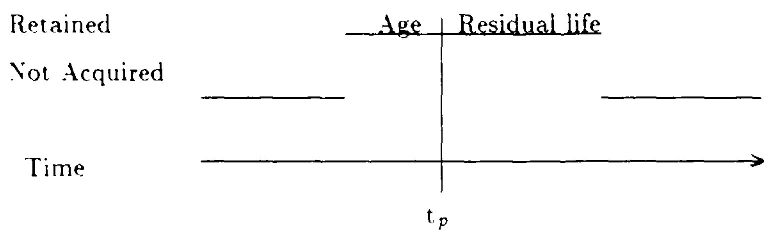

Figure 3. Example Showing Age and Residual Life of the Retention Time Distribution

The Movement State of the Relocatable Target

The target unit may be in one of two alternating states with respect to movement: it can be in the *move* state (moving), or it can be in the *stay* state (stationary). We define the random variable S to represent the length of time that a target is stationary and the random variable M to indicate the length of time that it is moving. We assume that all S and M are mutually independent and represent the unit movement as an alternating renewal process. If we examine the subunit state at any arbitrary point in time, it has a probability of being in the stay state (stationary) equal to p_{stay}.

$$p_{stay} = \frac{E[S]}{E[S] + E[M]}, \qquad [1]$$

where the notation E[X] denotes the expectation of the random variable X.

A target can be acquired and observed while either stationary or moving, so T_a and T_r may be dependent upon S and M. However, modeling acquisition at the theater level requires aggregation of acquisition data representing the net times to acquisition of more than 10,000 potential relocatable target units using all of the sensors deployed by the opposing force. The only theater-level data available at the present time is generated by a separate target acquisition model, TADER (Penn and Bauman [1987]), run at CAA. Given many characteristics of the sensors, target units, and battlefield scenario, to include the expectations of the stay and move times S and M, TADER output for each unit is used to estimate the parameters of the distributions of T_a and T_r. The effect of unit movement is averaged out in the TADER calculations and is not recoverable. As a result, the only distributional data available to support the determination of the joint distribution of T_a, T_r, S, and M are the univariate distributions. Therefore, in our model, we assume that if the fixed *parameters* of the distributions of T_a, T_r, S, and M are known, then the *variables* T_a, T_r, S, and M are mutually independent. This assumption of independence, as is usually the case, is a simplistic approximation to complex, poorly understood dependencies.

The assumption of independence between the processes of acquisition and movement is explicitly or implicitly present in all models and studies examining nuclear exchanges at the theater level (to include NUFAM III) of which the author is aware. At the high level of aggregation present in theater analyses, even more simplistic models are frequently used. For example, the issue of target retention is frequently ignored, and the movement of units is either ignored or represented in a less realistic manner. It is our intent to provide a simple alternative to detailed simulations, such as NUFAM III, that permits rapid calculation of the probabilities of acquisition and engagement of relocatable nuclear targets, without loss of precision. If the model presented herein is used in lieu of a model making similar or more simplified assumptions, this purpose is achieved.

Representing the Target Acquisition Process

When constructing a model of the acquisition and engagement of relocatable nuclear targets, it is not necessary to explicitly represent the target acquisition and movement processes in a detailed simulation. Since these processes can be represented as alternating renewal processes, a well-known result of renewal theory provides us the probability of acquisition, p_{acq}, at any random point of time:

$$p_{acq} = \frac{E[\ T_r\ ]}{E[\ T_a\ ] + E[\ T_r\ ]}\ . \qquad [2]$$

The time to acquisition, T_a, is normally derived from detailed sensor simulation models. The TADER model provides for each unit a probability p_a that it was acquired by any part of the sensor systems on a side during a short interval of time δ_t. These values p_a are fit to a standard "glimpse" type model of target acquisition, where there is a probability p_a that the unit is detected (acquired) during any time interval δ_t. Thus the probability of detecting the unit on the kth such glimpse is $p_a(1-p_a)^{k-1}$, k=1,2,... . This is geometric with:

$$P[\ T_a > t\ ] = (\ 1-p_a\ )^{\left\lfloor \frac{t}{\delta_t} \right\rfloor},\quad t \geq 0; \qquad E[\ T_a\ ] = \frac{\delta_t}{p_a}, \qquad [3]$$

where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

The length of time that the target unit is retained on our acquired target list, T_r, will be dependent upon our assumptions about the intelligence collection process and the units represented in the scenario. When the target unit cannot be observed after acquisition, we use a constant retention time equal to the expectation of the residual stay time: that is, $T_r = E[\ Y_s\ ]$. When the target unit is subject to periodic observations subsequent to the acquisition time, the period between observations is assumed to be the same as for the acquisitions (δ_t).

If the probability of continuing to detect the target unit during a single glimpse, given that it was detected on the previous glimpse, is p_r, then the probability that the target acquisition is lost during a single glimpse is ($1 - p_r$), and the retention time $T_r = \delta_t K_r$, where K_r is distributed as geometric($1 - p_r$). Obviously, p_r will be considerably higher than the probability of detection given that the unit was not detected previously. We handle the continuous observation cases by using the same glimpse model with a very short interval δ_t. Methods used at CAA for estimating T_r are discussed below.

The probability that the unit detected at time t will remain on the target list until some time s is simply the probability that the time remaining until the next transition point (dropping the unit from the list) after t will be greater than ($s - t$). If we enter a renewal process at some random point, we will be more likely to enter at a point where there is a long time between transitions rather than an average length of time. Thus the distribution of the residual life (time remaining until the next state transition) is not the same as the underlying interarrival distribution. Fortunately, the distribution for the residual life has been worked out (cf. Ross [1983], pg. 68). If we let Y_r denote the residual life of the retention time, renewal theory tells us that

$$P[\, Y_r > y \,] = 1 - \frac{1}{E[T_r]} \int_{u=0}^{y} P[\, T_r > u \,]\, du \,. \qquad [4]$$

Combining this probability with p_{acq} given above, the probability that a unit is acquired at time t and remains on the list for s additional hours is

P[on the acquisition list at time t and retained from t to $t + s$]

$$= \frac{E[\, T_r \,] \; - \; \int_{u=0}^{s} \overline{F}_{T_r}(u)\, du}{E[\, T_a \,] + E[\, T_r \,]} \,, \qquad [5]$$

where $\overline{F}_{T_r}(u)$ denotes $P[\, T_r > u \,]$. For additional details on implementing alternating renewal processes, see Youngren [1989].

The parameters of the distributions of T_a, T_r, S, and M are determined based on the theater-level scenario during the time period (in the case of FORCEM, a 12-hour time period) that includes the time t_p. We would expect that these parameters will change over time; however, these changes are generally gradual in our scenarios, and we are only interested in the process for a short period (less than 12 hours) around the time t_p. As a result, we assume that these parameters remain fixed.

An Example

Suppose that the probability p_a that a target unit is detected during an interval δ_t is equal to 0.2 with $\delta_t = 1$ hour. Then $E[\, T_a \,] = \frac{\delta_t}{p_a} = 5$ hours.

Suppose that the stay time S is distributed as uniform (10 min., 40 min.) and the move time M is distributed as uniform (90 min., 120 min.). Then E[S] = 25 min. and E[M] = 105 min. From this, we can calculate the probability that the unit is stationary at any random point in time, p_{stay}:

$$p_{stay} = \frac{25}{25 + 105} = 0.192.$$

We can also calculate the distribution of the residual stay time, Y_s:

$$P(\ Y_s > y\ ) = 1 - \frac{1}{E[S]}\int_{u=0}^{y} P[\ S > u\ ]\ du$$
$$= 1 - \frac{y}{25} \qquad 0 \leq y < 10 \text{ min.}$$
$$= \frac{1}{75}(\ 80 - 4y + \frac{y^2}{20}\ ) \qquad 10 \text{ min.} \leq y < 40 \text{ min.}$$

The expectation of the residual stay time, $E[\ Y_s\ ] = \int_{y=0}^{40} P[\ Y_s > y\ ]\ dy = 14$ min.

Determining the Probability that a Unit is Available for Fire

In order for a unit to be available for fire by nuclear weapons, it must be acquired and retained as a target at least until the scheduled firing time. The target is evaluated at some known planning time t_p; during the planning period, the detonation will be scheduled to occur at some time T_d within the fire execution period. Thus a target that is on the acquisition list at time t_p must be retained at least until T_d; this occurs when the residual time on the acquisition list, Y_r, is greater than or equal to $(\ T_d - t_p\ )$ (Figure 2). We define the probability P[unit is acquired and retained until detonation] as p_{avail}. Thus $p_{avail} = P[\ Y_r > T_d - t_p \mid \text{acquired at } t_p\ ] \cdot P[\ \text{acquired at } t_p\ ]$.

We calculate p_{avail} using the residual life of the retention time. In order to make the development complete, we define R as the maximum permitted value for T_r and L as the maximum permitted value for $(T_d - t_p)$; $0 < R < \infty$; $0 < L < \infty$. For convenience we use the notation "$P[\ X = x\ ]$" to denote the probability density function $f_X(x)$ of a continuous random variable. Thus (conditioned on being on the acquisition list at time t_p) we have:

$$P[\ Y_r > T_d - t_p \mid T_d - t_p = t\ ] = 1 - \frac{1}{E[T_r]}\int_{u=0}^{t} \bar{F}_{T_r}(u)\ du \qquad 0 \leq t < R,$$
$$= 0 \qquad t \geq R,$$

and

$$P[\ Y_r > T_d - t_p\ ] = \int_{t=0}^{D} P[\ Y_r > T_d - t_p \mid T_d - t_p = t\ ]\ P[\ T_d - t_p = t\ ]\ dt.$$

Thus

$$p_{avail} = P[\ Y_r > T_d - t_p \mid \text{acquired at time } t_p\ ] \cdot P[\ \text{acquired at } t_p\ ]$$

$$= \left[\int_{t=0}^{D} \left\{ 1 - \frac{1}{E[T_r]} \int_{u=0}^{t} \bar{F}_{T_r}(u)\ du \right\} \cdot f_{T_d - t_p}(t)\ dt \right] \cdot \frac{E[\ T_r\ ]}{E[\ T_a\ ] + E[\ T_r\ ]} \qquad [6]$$

where $D = \min\{\ R, L\ \}$; $0 \leq T_d - t_p \leq L < \infty$; $0 \leq Y_r \leq R < \infty$; $T_r \geq 0$; $t_p \geq 0$.

In some cases (such as *a* and *b* below), we also require that the unit be stationary at the last time that it was observed. When this occurs, P_{avail} is multiplied by the probability that the unit was stationary, p_{stay}.

Determining the Probability of Hitting a Relocatable Nuclear Target

A unit can be *hit* if it is available for fire and is stationary at time T_d at the place where it was last observed. We do not consider the availability of a suitable weapon, in range, that can engage the target unit, nor do we assess the effects of the detonation, taking into account delivery system accuracy, etc. The calculation of the probability of hit is designed to tell us what our *opportunity* for hitting a relocatable target might be. The precise calculations of the probability of hit depend upon the capabilities of the sensor system(s) to observe the target after acquisition. Four cases that have been used at CAA are discussed below.

Case a. No Capability Exists to Observe the Target after Acquisition

This case represents the situation when there is no opportunity for any sensor to observe the target between the acquisition time and the end of fire planning. This may occur when a potential target is observed by some sort of sensor (e.g., aerial reconnaissance) with a single mission over the area, when the period between observation opportunities is long with respect to the planning and firing process, or when the target is able to conceal itself to the extent that subsequent sensor missions cannot verify either the presence or absence of the target. In this case, the target remains on the acquisition list until there is a perception that the information is too "old" to plan a nuclear fire. The time interval that defines "old" will be a matter of judgment and/or doctrine, and is generally based on an estimate of the time that the acquired unit will remain in place: e.g., the expected residual life of the stay time.

In order for the acquisition to be useful, the target unit must have been stationary at the time of acquisition. The joint probability that the unit is on the acquisition list at the planning time t_p and was stationary at the acquisition time A is:

P[unit on the acquisition list at t_p and stationary at A]

$$= p_{acq} \cdot p_{stay} = \frac{E[\, T_r \,]}{E[\, T_a \,] + E[\, T_r \,]} \cdot \frac{E[\, S \,]}{E[\, S \,] + E[\, M \,]} \cdot \qquad [7]$$

The probability that the unit is available for fire is:

$$p_{avail} = P[\text{ unit retained until } T_d, \text{ on the acquisition list at } t_p, \text{ and stationary at A }]$$
$$= P[\ Y_r > T_d - t_p \mid \text{on acquisition list at } t_p \text{ and stationary at A }]$$
$$\cdot\ P[\text{ unit on the acquisition list at } t_p\ ] \cdot P[\text{ unit stationary at A }]\text{, thus}$$

$$p_{avail} = \left[\int_{t=0}^{D} P[\ Y_r > T_d - t_p \mid T_d - t_p = t\ ]\ P[\ T_d - t_p = t\ ]\ dt \right]$$
$$\cdot\ \frac{E[\ T_r\ ]}{E[\ T_a\ ] + E[\ T_r\ ]} \cdot \frac{E[\ S\ ]}{E[\ S\ ] + E[\ M\ ]}\,. \qquad [8]$$

where $D = \min\{\ R, L\ \}$; $0 \le T_d - t_p \le L < \infty$; $0 \le Y_r \le R < \infty$; $T_r \ge 0$; $A \ge 0$.

Finally, the probability of hit, p_{hit}, is the joint probability that the unit is available for fire and will still be stationary at time T_d at the location at which it was acquired. The acquisition time A is not known, but we know that the difference ($t_p - A$) is distributed as the age of the distribution of the time of acquisition, Y_r (the age and the residual life are identically distributed), independent of the distribution of $T_d - t_p$. Because of independence, p_{hit} is calculated as:

$$p_{hit} = P[\ Y_r > T_d - t_p \mid \text{on acquisition list at } t_p\ ] \cdot P[\text{ unit on the acquisition list at } t_p\ ]$$
$$\cdot\ P[\ Y_S > T_d - A \mid \text{stationary at A }] \cdot P[\text{ unit stationary at A }]\,.$$

$$= \left[\int_{t=0}^{D} P[\ Y_r > T_d - t_p \mid T_d - t_p = t\ ]\ P[\ T_d - t_p = t\ ]\ dt \right] \cdot \frac{E[\ T_r\ ]}{E[\ T_a\ ] + E[\ T_r\ ]}$$

$$\cdot \int_{y=0}^{\infty} \int_{t=0}^{D^*} P[Y_s > (T_d - t_p) + (t_p - A) \mid T_d - t_p = t \text{ and } (t_p - A) = y]\ P[T_d - t_p = t]\ P[t_p - A] = y]\ dt\ dy$$

$$\cdot\ \frac{E[\ S\ ]}{E[\ S\ ] + E[\ M\ ]}\,. \qquad [9]$$

where $D^* = \min\{\ L, S_u\ \}$; $S_u = \max\{\ S\ \}$; $0 < S \le S_u < \infty$.

<u>*Example Case a*</u>

For this example, we set the retention time $T_r = E[\ Y_s\ ] = 14$ minutes, a constant; thus $P[T_r > t] = 1$ for $0 \le t \le E[\ Y_s\ ]$; 0 for $t > E[\ Y_s\ ]$. Obviously $E[\ T_r\ ] = E[\ Y_s\ ] = 14$ minutes.

Recalling that $E[\, T_d \,] = 5$ hours,

$$p_{acq} = \frac{14}{(5 \cdot 60) + 14} = 0.045.$$ The residual retention time Y_r is distributed as:

$$P[\, Y_r > y \,] = 1 - \frac{1}{E[T_r]} \int_{u=0}^{y} P[\, T_r > u \,]\, du = 1 - \frac{y}{R} \qquad 0 \leq y \leq R; \ 0 \text{ for } y > R,$$

where $R = \max\{\, T_r \,\} = 14$ min.

We assume that the detonation time is equally likely to occur during the fire execution period from t_p to $t_p + L$, with $L=60$ min. Thus $(\, T_d - t_p \,) \sim$ uniform $(\, 0, 60$ min. $)$

$$p_{avail} = \left[\int_{t=0}^{R} P[\, Y_r > T_d - t_p \mid T_d - t_p = t \,]\, P[\, T_d - t_p = t \,]\, dt \right]$$

$$\cdot \frac{E[\, T_r \,]}{E[\, T_a \,] + E[\, T_r \,]} \cdot \frac{E[\, S \,]}{E[\, S \,] + E[\, M \,]}$$

$$= \left[\int_{t=0}^{R} [\, 1 - \frac{t}{R} \,] \cdot \frac{1}{60}\, dt \right] \cdot p_{acq} \cdot p_{stay} = 0.117 \cdot 0.045 \cdot 0.192 = 0.001$$

To calculate p_{engage}, we recall that $(\, t_p - A \,)$ is distributed as the age of the retention time (identical to the distribution of the residual life Y_r). Thus

$$p_{engage} = p_{avail} \cdot$$

$$\int_{y=0}^{\infty} \int_{t=0}^{D} P[Y_s > (T_d - t_p) + (t_p - A) \mid T_d - t_p = t \text{ and } (t_p - A) = y]\, P[T_d - t_p = t]\, P[t_p - A = y]\, dt\, dy.$$

After some messy algebra,

$$p_{engage} = 0.001 \cdot 0.138 \simeq 0.0001 .$$

Case b. The Target is Observed Periodically after Acquisition

This case represents the situation when there is an opportunity for at least one sensor to observe the target periodically (but not continuously) after acquisition. This may occur when a potential target is observed by some sort of sensor which can observe the area at periodic intervals. In this case, we assume that the target remains on the acquisition list until a negative sensor report is received. Note that this may occur either because a unit has moved or the sensor failed to detect the unit on a subsequent observation. The probability of detecting the target unit on a relook will normally be considerably higher than the initial probability of detection.

We define a time T_l as the last time that the target unit was observed prior to t_p; thus $T_l \leq t_p < T_d$. The analysis from this point onward is straightforward; except for the distribution of $(t_p - T_l)$, all of the equations in the previous section hold with T_l replacing A.

$$p_{avail} = \left[\int_{t=0}^{D} P[\, Y_r > t_d - t_p \mid T_d - t_p = t \,]\, P[\, T_d - t_p = t \,]\, dt \right]$$

$$\cdot \frac{E[\, T_r \,]}{E[\, T_a \,] + E[\, T_r \,]} \cdot \frac{E[\, S \,]}{E[\, S \,] + E[\, M \,]} \,. \qquad [10]$$

Finally, the probability of hit, p_{hit}, is the joint probability that the unit was stationary at the time of acquisition A, it is on the acquisition list at time t_p, it will be retained as a target at least until time T_d, and it will still be stationary at time T_d at the location at which it was last observed. The time T_l is not known, but we know that time t_p occurs at a random point during the interval between observations. We can regard the interval $(t_p - T_l)$ as distributed as the age of the distribution of the time between observations, as long as the interobservation times are iid. We have examined the simple case where the interobservation times are constant at δ_t; thus, $t_p - T_l$ is distributed as uniform $(0, \delta_t)$.

We see that p_{hit} is calculated as:

$$p_{hit} = P[\, Y_r > T_d - t_p \text{ and } Y_S > T_d - T_l \mid \text{on acquisition list at } t_p \text{ and stationary at } T_l \,]$$

$$\cdot\, P[\, \text{unit on the acquisition list at } t_p \,] \cdot P[\, \text{unit stationary at } T_l \,] \,,$$

$$= \left[\int_{t=0}^{D} P[\, Y_r > T_d - t_p \mid T_d - t_p = t \,]\, P[\, T_d - t_p = t \,]\, dt \right] \frac{E[\, T_r \,]}{E[\, T_a \,] + E[\, T_r \,]}$$

$$\cdot \int_{y=0}^{\delta_t} \int_{t=0}^{D^*} P[Y_s > (T_d - t_p) + (t_p - T_l) \mid T_d - t_p = t;\ (t_p - T_l) = y]\ P[T_d - t_p = t]\ P[t_p - T_l = y]\ dt\, dy$$

$$\cdot \frac{E[\, S \,]}{E[\, S \,] + E[\, M \,]} \,. \qquad [11]$$

Example Case b

Suppose that the probability of detecting the target during any glimpse period of length δ_t, given that it was detected during the previous glimpse (p_r), is 0.9. Thus the probability that we *fail* to detect the unit on the kth glimpse (and therefore drop it from our acquisition list) is equal to $(1 - p_r)\, {p_r}^{k-1}$, $k = 1, 2, \ldots$.

As a result, the distribution of the retention time T_r is:

$$P[\, T_r > t \,] = P[\, K_r > \left\lfloor \frac{t}{\delta_t} \right\rfloor], \quad t \geq 0$$

where $K_r \sim$ geometric ($1-p_r$), and $E[\, T_r \,] = \frac{\delta_t}{1-p_r} = \frac{1 \text{ hour}}{1 - 0.9} = 10$ hours. Recalling from the previous example that $E[\, T_a \,] = 5$ hours, $p_{acq} = \frac{10}{10 + 5} = 0.667$.

Calculating the residual life (or age) of the retention time is straightforward. We define an integer $m \geq 0$ such that $m\delta_t \leq y < (m+1)\delta_t$ for any $y \geq 0$.

$$
\begin{aligned}
P[\, Y_r > y \,] &= 1 - \frac{1-p_r}{\delta_t} \int_{u=0}^{y} P[\, T_r > u \,]\, du \\
&= 1 - \frac{1-p_r}{\delta_t} \left[\sum_{k=0}^{m-1} \int_{u=m\delta_t}^{(m+1)\delta_t} P[\, T_r > u \,]\, du + \int_{u=m\delta_t}^{y} P[\, T_r > u \,]\, du \right] \\
&= 1 - \frac{1-p_r}{\delta_t} \left[\delta_t \sum_{k=0}^{m-1} p_r{}^k + (y - m\delta_t)\, p_r{}^m \right] \\
&= 1 - (1-p_r) \frac{p_r{}^m - 1}{p_r - 1} - \frac{1-p_r}{\delta_t} (y - m\delta_t)\, p_r{}^m \\
&= \left[1 - \frac{1-p_r}{\delta_t} (y - m\delta_t) \right] p_r{}^m , \qquad m\delta_t \leq y < (m+1)\delta_t, \;\; m \geq 0.
\end{aligned}
$$

In order to calculate p_{avail}, we need to determine $D = \min\{ R, L \}$, where $R = \max\{ T_r \} \to \infty$ and $L = \max\{ T_d - t_p \} = \delta_t = 1$ hour. Then

$$p_{avail} = \left[\int_{t=0}^{D} P[\, Y_r > T_d - t_p \mid T_d - t_p = t \,]\, P[\, T_d - t_p = t \,]\, dt \right] \cdot p_{acq} \cdot p_{stay}$$

$$p_{avail} = 0.95 \cdot p_{acq} \cdot p_{stay} = 0.95 \cdot 0.667 \cdot 0.192 = 0.122 \,.$$

We define a time T_l as the last time that the target unit was observed prior to the time of end of the planning process, t_p; thus $T_l \leq t_p < T_d$. Since δ_t is constant at 1 hour, ($t_p - T_l$) $\sim$ uniform(0, 60 min.). $D^* = \min\{ L, S_u \} = \min\{ 60$ min., 40 min. $\} = 40$ min.

$$p_{engage} = p_{avail} \cdot \int_{y=0}^{60} \int_{t=0}^{40} P[Y_s > (T_d - t_p) + (t_p - T_l) \mid T_d - t_p = t;\, t_p - T_l = y]\, P[T_d - t_p = t]\, P[t_p - T_l = y]\, dt\, dy$$

After some messy algebra, $p_{engage} = 0.122 \cdot 0.039 = 0.005$.

Case c. The Target is Observed Continuously after Acquisition with Preplanned Fire

This case represents the situation when at least one sensor can observe or "track" the target continuously (or nearly continuously) after acquisition. This may occur when a potential target is observed by a fixed sensor that holds the target unit in view (e.g., radar). In this case, we assume that the target remains on the acquisition list unit until the sensor loses the target due to movement out of range or into concealment. or until something breaks the tracking, such as obscuration, etc. The assumption of independence between the acquisition and movement processes breaks down when continuous observation is possible. However, the simulation model (NUFAM III) and the acquisition data currently available make this assumption, so we carry it forward into our probability model and interpret the results accordingly. CAA is in the process of acquiring new data that will enable us to estimate the joint distributions directly (see section on future research).

In this case. the acquisition is useful if the target unit was stationary or moving at the time of acquisition. The probability that the unit is on the acquisition list at the planning time t_p is:

$$\text{P[unit on the acquisition list at } t_p \text{]} = p_{acq} = \frac{E[\, T_r \,]}{E[\, T_a \,] + E[\, T_r \,]} \qquad [12]$$

The probability that the unit is available for fire is:

P[$Y_r > T_d - t_p$ | on acquisition list at t_p] · P[unit on the acquisition list at t_p], thus

$$p_{avail} = \left[\int_{t=0}^{D} P[\, Y_r > T_d - t_p \mid T_d - t_p = t \,]\, P[\, T_d - t_p = t \,]\, dt \right] \cdot \frac{E[\, T_r \,]}{E[\, T_a \,] + E[\, T_r \,]}. \qquad [13]$$

Since we can track the target unit. it need not be stationary at the time of acquisition. but it must be stationary at the time our fire planning is complete (t_p) and remain at that location until detonation (T_d). The time of detonation T_d continues to be fixed during the time of fire planning. The probability of hit, p_{hit}, is the joint probability that the unit is available for fire and will be stationary from the time t_p to T_d. The times t_p and T_d are preplanned (independent, by assumption, of the stay/move process), so we can simply multiply by p_{stay} to get the probability that the unit is stationary at time t_p.

$$\begin{aligned} p_{hit} = {} & \text{P[} Y_r > T_d - t_p \text{ | on acquisition list at } t_p \text{]} \cdot \text{P[unit on the acquisition list at } t_p \text{]} \\ & \cdot \text{P[} Y_s > T_d - t_p \text{| stationary at } t_p \text{]} \cdot \text{P[unit stationary at } t_p \text{]}. \end{aligned} \qquad [14]$$

$$p_{hit} = \left[\int_{t=0}^{D} P[\, Y_r > T_d - t_p \mid T_d - t_p = t \,]\, P[\, T_d - t_p = t \,]\, dt \right] \cdot \frac{E[\, T_r \,]}{E[\, T_a \,] + E[\, T_r \,]}$$

$$\cdot \left[\int_{t=0}^{D^*} P[\, Y_s > T_d - t_p \mid T_d - t_p = t \,]\, P[\, T_d - t_p = t \,]\, dt \right] \cdot \frac{E[\, S \,]}{E[\, S \,] + E[\, M \,]} \qquad [15]$$

Example Case c

Currently, we model a nearly continuous observation using the same glimpse model used in the previous sections, with a short glimpse period $\delta_t = 5$ min. If the probabilities of detection p_a and retention $(1\text{-}p_r)$ are the same as given in the previous examples,

$$E[\, T_a \,] = \frac{\delta_t}{p_a} = 25 \text{ min., } E[\, T_r \,] = \frac{\delta_t}{1 - p_r} = 50 \text{ min., and } p_{acq} = 0.667 \,.$$

$$p_{avail} = \left[\int_{t=0}^{60} P[\, Y_r > T_d - t_p \mid T_d - t_p = t \,]\, P[\, T_d - t_p = t \,]\, dt \right] \cdot \frac{E[\, T_r \,]}{E[\, T_a \,] + E[\, T_r \,]} \,.$$

$$= \left[\delta_t \sum_{k=0}^{11} {p_r}^k \right] \cdot p_{acq} = \left[\delta_t \left[\frac{{p_r}^{12} - 1}{p_r - 1} \right] \right] \cdot p_{acq} = 0.600 \cdot 0.667 = 0.400 \,.$$

$$p_{engage} = p_{avail} \cdot \int_{t=0}^{40} P[\, Y_s > T_d - t_p \mid T_d - t_p = t \,]\, P[\, T_d - t_p = t \,]\, dt$$

$$\cdot \frac{E[\, S \,]}{E[\, S \,] + E[\, M \,]} = 0.400 \cdot 0.233 \cdot 0.192 = 0.018 \,.$$

Case d. The Target is Observed Continuously after Acquisition without Preplanned Fire

This case represents the same situation as case *c* where at least one sensor can observe or "track" the target continuously (or nearly continuously) after acquisition. However, it is reasonable to suppose that if we can track the target unit continuously, we would give the order to fire when unit stops for the first time within the fire execution period. Until now, we have given the order to fire (determined the time T_d) at time t_p. In this case, we define a (random) time T_f during the fire execution period of length L which depends upon the time that the unit stops. If the weapon can be fired immediately upon detection that the unit has stopped, then (ignoring the time of flight of the round) $T_d = T_f$. However, it is more reasonable to assume that some time will have to be spent getting the order to the delivery unit, adjusting the aimpoint to the location at which the unit stopped, confirming that the location meets the criteria for employment of the weapon, etc. We denote this time $(T_d - T_f)$ as η, and for simplicity, we assume that η is constant.

If the unit is stationary at time t_p, $T_f = t_p$; otherwise, T_f equals the first time the unit stops between t_p and $t_p+(L-\eta)$. If the move time distribution is not bounded from above at some value less than the fire execution period duration, there is a positive probability that the unit will be moving during the entire fire execution period. The hit probabilities can be adjusted to account for this.

Because the stay/move process is assumed independent of the acquisition process, T_f is random with respect to the acquisition process. Therefore, the probability that the unit is on the acquisition list at the time T_f is simply p_{acq}.

The probability that the unit is available for fire is still the probability that it was on the acquisition list at T_f and retained between T_f and T_d. However, if we issue the fire order at time T_f, we will retain the unit between T_f and T_d by definition. Thus, the conditional probability P[unit retained until T_d | unit on the acquisition list at T_f] = 1. As a result,

$$p_{avail} = 1 \cdot p_{acq} \qquad [16]$$

The probability that the unit will be stationary at the time t_p is simply p_{stay}. The probability that the unit stops within the period (t_p to $t_p+(L-\eta)$), given that it was moving at time t_p, is equal to the probability that residual life of the move time, Y_m, is less than $(L-\eta)$.

The probability of hit, p_{hit}, is therefore the joint probability that the unit is on the acquisition list at the stopping time T_f, it will be retained as a target at least until time T_d, and it will be stationary from time T_f to time T_d. If $T_f = t_p$, the unit was somewhere in its stay period at time t_p, and the probability that it will remain in place until T_d is $P[\, Y_s > \eta \,]$, where Y_s is the residual life of the stay time. If $t_p < T_f < L-\eta$, then the unit stopped during the fire execution period and T_f is the beginning of the stay time. Thus $P[\text{ stationary at } T_d \mid t_p < T_f < L-\eta\,] = P[\, S > \eta \,]$.

$$\begin{aligned} p_{hit} &= P[\, Y_r > T_d - T_f \mid \text{on acquisition list at } T_f \,] \cdot P[\text{ unit on the acquisition list at } T_f \,] \\ &\quad \cdot \Big\{ P[\, Y_s > \eta \mid \text{stationary at } t_p \,] \cdot p_{stay} \\ &\qquad + P[\, S > \eta \mid \text{stopped at } T_f \,] \cdot P[\, Y_m < L-\eta \mid \text{moving at } t_p \,] \cdot p_{move} \Big\} \\ &= p_{acq} \cdot \Big[\, P[\, Y_s > \eta \,] \cdot p_{stay} + P[\, S > \eta \,] \cdot P[\, Y_m < L-\eta \,] \cdot p_{move} \,\Big] \end{aligned}$$

$$\text{where } p_{acq} = \frac{E[\, T_r \,]}{E[\, T_a \,] + E[\, T_r \,]} \text{ and } p_{stay} = \frac{E[\, S \,]}{E[\, S \,] + E[\, M \,]} . \qquad [17]$$

Example Case d

From the above, $p_{avail} = 1 \cdot p_{acq} = 1 \cdot 0.667 = 0.667$

Suppose that the time $(T_d - T_p)$, η, is equal to 15 min. The distribution of the residual move time, Y_m, is computed the same way as the distribution of Y_s.

$P[\, Y_m > y \,] = 1 - \frac{1}{E[M]} \int_{u=0}^{y} P[\, M > u \,]\, du$. For $0 \leq y \leq 105$ min., $P[\, Y_m > y \,] = 1 - \frac{y}{105}$.

$$p_{engage} = p_{acq} \cdot \left[\, P[\, Y_s > \eta \,] \cdot p_{stay} + P[\, S > \eta \,] \cdot P[\, Y_m < L - \eta \,] \cdot p_{move} \,\right]$$

$$= 0.667 \cdot \left[\, 0.417 \cdot 0.192 + \frac{25}{30} \cdot \frac{45}{105} \cdot 0.808 \,\right] = 0.246 \, .$$

Modeling Subsequent Laydowns

Up to now, all of the development has been centered upon a single nuclear laydown per side, which involves a single planning period and single reference point t_p at which we determine the unit acquisition and movement states and the associated residual life distributions. If we wish to model subsequent laydowns, we will need to make determinations of the unit states at some later planning time, say t_p^*, where $t_p^* > t_p$. If t_p^* and t_p are sufficiently far apart (this is a matter of judgement -- generally at least the expected length of several transition cycles), then the state determination for each unit at time t_p^* can be treated as independent of the state determination at time t_p. If t_p^* is close in time to t_p, the state determinations will be *dependent* and this dependency should be represented in the model. For a discussion of these points, along with a suggested approximation for this dependency relationship, see Youngren [1989].

Applications of this Methodology at CAA

The US Army Concepts Analysis Agency uses the methods described previously to determine the probability that relocatable target units may be acquired and hit by tactical nuclear weapons at times when running the detailed simulation model NUFAM III is not possible or desirable. These methods permit an estimation of the impact of varying input data (for example, different values for the average stay and move times for units) on NUFAM output, without running the NUFAM model. These same methods can be incorporated into NUFAM III to replace the explicit simulation of the acquisition and movement events. Such a replacement eliminates over 2 million simulation events per NUFAM excursion without a loss of relative accuracy in the results of interest.

Future Research

To eliminate the necessity of assuming independence between acquisition and movement, and to improve our estimates of acquisition and the probability of hitting the target, CAA has an ongoing data collection effort with the US Army Systems Analysis Agency to try to determine the probabilities of acquisition and retention under multiple factors such as stationary/moving; posture; weather; obscuration; sensor system deployment, etc. Detailed sensor models will be used to estimate the distribution functions for T_a and T_r conditioned on these factors. The Agency's target acquisition model will be modified to vary the factors according to their underlying distributions in order to estimate the unconditional joint distributions of T_a, T_r, S, and M for the scenario of interest. These distributions will be used to improve our estimates of acquisition and engagement and to refine the probability model to account for the dependencies between movement and acquisition.

The probability models discussed in this paper will eventually be incorporated into a new stochastic model of a theater-level nuclear exchange called NEMESIS, which will replace NUFAM III. NEMESIS will combine the probabilistic description of our uncertainty about the high resolution processes of target acquisition and movement given in this paper with a probabilistic description of our uncertainty about the locations of the potential target units, providing increased resolution in space as well as time over theater models such as FORCEM. The NEMESIS model may either be used as a stand-alone model of theater nuclear warfare (in a manner similar to NUFAM III) or as a means of generating realizations of possible nuclear exchange outcomes for input into the FORCEM model. The latter use permits us to model the effect of different possible nuclear exchange outcomes on subsequent conventional battle in FORCEM.

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